Period 1, Sept 18, 2024

In Exercises 55–58, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

Function	Point
55. $y = x^4 - 3x^2 + 2$	$(1, 0)$
56. $y = x^3 + x$	$(-1, -2)$

$F(x) = y = a x^n$

$F'(x) = \frac{dy}{dx} = a \cdot n \cdot x^{n-1}$

29. $y = \frac{\sqrt{x}}{x} = \frac{x^{1/2}}{x^1} = x^{\frac{1}{2}-1} = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2} \cdot x^{-\frac{1}{2}-1} = -\frac{1}{2} x^{-\frac{3}{2}} = \boxed{\frac{-1}{2x^{3/2}}} = \frac{-1}{2x\sqrt{x} \cdot \sqrt{x}} \sqrt{x} = \frac{-\sqrt{x}}{2x \cdot x} = \boxed{\frac{-\sqrt{x}}{2x^2}}$

33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3$ $\quad (0, -\frac{1}{2})$ Plug in

$F'(x) = 0 + \frac{7}{5} \cdot 3 \cdot x^{3-1} \Rightarrow F'(x) = \frac{21}{5} \cdot x^2 \Rightarrow F'(0) = \frac{21}{5}(0)^2 = 0$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

45. $f(x) = \frac{x^3 - 3x^2 + 4}{x^2} = \frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{4}{x^2} = x^1 - 3 + 4x^{-2}$

$F'(x) = 1 \cdot x^{1-1=0} - 0 + 4 \cdot -2 \cdot x^{-2-1} = 1 \cdot 1 - 0 - 8x^{-3} = \frac{1x^3}{1x^3} - \frac{8}{x^3} = \frac{x^3}{x^3} - \frac{8}{x^3} = \frac{x^3 - 8}{x^3} = \frac{x^3 - 2^3}{x^3}$

$\frac{(x-2)(x^2 + 2x + 4)}{x^3}$

37. $f(\theta) = 4\sin\theta - \theta$ $\quad (0, 0)$

$F'(\theta) = 4 \cdot \cos\theta - 1$

$F'(0) = 4 \cdot \cos 0 - 1 = 4 \cdot 1 - 1 = 4 - 1 = 3$

43. $f(x) = \frac{4x^3 + 3x^2}{x} = \frac{4x^3}{x} + \frac{3x^2}{x} = 4x^2 + 3x$

$F'(x) = 4 \cdot 2 \cdot x^{2-1} + 3 \cdot 1 \cdot x^{1-1} = 8x^1 + 3x^0 = 8x^1 + 3 \cdot 1 = 8x + 3$

47. $y = x(x^2 + 1) = x^3 + x$

$y = x^3 + x^1 \Rightarrow \frac{dy}{dx} = 3 \cdot x^{3-1} + 1 \cdot x^{1-1} = 3x^2 + x^0 = 3x^2 + 1$

In Exercises 31–38, find the slope of the graph of the function at the given point. Use the *derivative* feature of a graphing utility to confirm your results.

Function	*Point*
31. $f(x) = \frac{8}{x^2}$	$(2, 2)$
32. $f(t) = 3 - \frac{3}{5t}$	$(\frac{3}{5}, 2)$
33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3$	$(0, -\frac{1}{2})$
34. $y = 3x^3 - 10$	$(2, 14)$
35. $y = (4x + 1)^2$	$(0, 1)$
36. $f(x) = 3(5 - x)^2$	$(5, 0)$

31.

$F(x) = \frac{8}{x^2} = 8x^{-2}$

$F'(x) = 8 \cdot -2 \cdot x^{-2-1} = -16x^{-3} = -\frac{16}{x^3}$

$F'(2) = -\frac{16}{2^3} = -\frac{16}{8} = -2$

35. $y = (4x+1)^2 = 16x^2 + 8x + 1$

$\frac{dy}{dx} = 16 \cdot 2 \cdot x^{2-1} + 8 \cdot 1 \cdot x^{1-1} + 0$

$\frac{dy}{dx} = 32x + 8$

$32(0) + 8 = 8$

36

$F(x) = 3(5-x)^2$

$F(x) = 3(25 - 10x + x^2)$

$F(x) = 75 - 30x + 3x^2$

$F'(x) = 0 - 30 + 6x$

$F'(5) = -30 + 6(5) = -30 + 30 = 0$

13. $f(t) = -2t^2 + 3t - 6$

$F'(T) = -2 \cdot 2 \cdot T^{2-1} + 3 \cdot 1 \cdot T^{1-1} - 0$

$F'(T) = -4T + 3 \cdot T^0 = -4T + 3$

17. $s(t) = t^3 + 5t^2 - 3t + 8$

$S'(T) = 3 \cdot T^{3-1} + 5 \cdot 2 \cdot T^{2-1} - 3 \cdot 1 T^{1-1} + 0$

$S'(T) = 3T^2 + 10T - 3$

27. $y = \frac{6}{(5x)^3} = \frac{6}{125x^3} = \frac{6}{125} \cdot x^{-3}$

$\frac{dy}{dx} = \frac{6}{125} \cdot -3 \cdot x^{-3-1} = \frac{-18}{125} \cdot x^{-4} = \frac{-18}{125x^4}$

32. $f(t) = 3 - \frac{3}{5t}$ $\quad (\frac{3}{5}, 2)$

$F(T) = 3 - \frac{3}{5} \cdot T^{-1}$

$F'(T) = 0 - \frac{3}{5} \cdot -1 \cdot T^{-1-1} = \frac{3}{5}T^{-2} = \frac{3}{5T^2} = \frac{3}{5(\frac{3}{5})^2} = \frac{3}{\cancel{5} \cdot \frac{9}{\cancel{25}\,5}} = \frac{\frac{3}{1}}{\frac{9}{5}}$

$\frac{3}{1} \cdot \frac{5}{9} = \frac{15}{9} = \frac{5 \cdot \cancel{3}}{3 \cdot \cancel{3}} = \frac{5}{3}$

In Exercises 55–58, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

	Function	*Point*
55.	$y = x^4 - 3x^2 + 2$	$(1, 0)$
56.	$y = x^3 + x$	$(-1, -2)$
57.	$f(x) = \frac{2}{\sqrt[4]{x^3}}$	$(1, 2)$
58.	$y = (x^2 + 2x)(x + 1)$	$(1, 6)$

55.

$y = x^4 - 3x^2 + 2$ $\quad (1, 0)$

$\frac{dy}{dx} = 4x^3 - 6x$

$4(1)^3 - 6(1) = 4 - 6 = -2$

-2 = Slope

Point $(1,0)$ Plug in

$y = -2x + b$

$0 = -2(1) + b$

$2 = b$

$y = -2x + 2$

56. $y = x^3 + x$ $\quad (-1, -2)$

$\frac{dy}{dx} = 3x^2 + 1$

$m = 3(-1)^2 + 1 = 3 \cdot 1 + 1 = 4$

$y = 4x + b$ Plug in

$-2 = 4(-1) + b \Rightarrow -2 = -4 + b$

$b = 2$

$y = 4x + 2$

57. $f(x) = \frac{2}{\sqrt[4]{x^3}} = 2 \cdot x^{-\frac{3}{4}}$ (1, 2)

$F'(x) = 2 \cdot -\frac{3}{4} x^{-\frac{3}{4}-1} = -\frac{6}{4} x^{-\frac{7}{4}} = \frac{-3}{2\sqrt[4]{x^7}}$

$F'(1) = \frac{-3}{2\sqrt[4]{1^7}} = \frac{-3}{2}$

$y = -\frac{3}{2}x + b$ ← plug in

$2 = -\frac{3}{2}(1) + b$

$\frac{4}{2} = -\cancel{\frac{3}{2}} + b \Rightarrow b = \frac{7}{2}$ → $y = -\frac{3}{2}x + \frac{7}{2}$

$+\frac{3}{2}$ $\quad +\cancel{\frac{3}{2}}$

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78. Show that the graphs of the two equations $y = x$ and $y = 1/x$ have tangent lines that are perpendicular to each other at their point of intersection.

Slope or $\frac{a}{b} \Rightarrow -\frac{b}{a}$

$y = x$ and $y = \frac{1}{x}$

intersect

$x \cdot x = \frac{1}{\cancel{x}} \cancel{\cdot x}$

$x^2 = 1$

$x = (1, -1)$

$y = x$

$\frac{dy}{dx} = 1$

$y = 1x + b$

Point (1,1)

$1 = 1 \cdot 1 + b$

$b = 0$

$y = 1x$

$y = \frac{1}{x} = 1 \cdot x^{-1}$

$\frac{dy}{dx} = -1x^{-1-1} = -1x^{-2} = \frac{-1}{x^2}$

Slope at $x = 1$ and $x = -1$

$\frac{-1}{(1)^2} = -1$ and $\frac{-1}{(-1)^2} = -1$

Point (1,1) $m = -1$

$y = -1x + b$

$1 = -1 \cdot 1 + b$

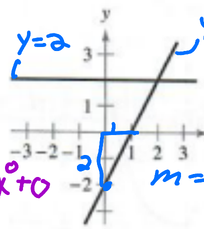
$2 = b$

$y = -x + 2$

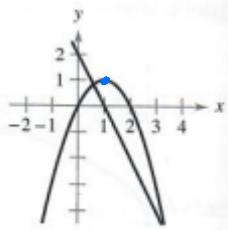
WRITING ABOUT CONCEPTS (continued)

In Exercises 75 and 76, the graphs of a function f and its derivative f' are shown on the same set of coordinate axes. Label the graphs as f or f' and write a short paragraph stating the criteria you used in making your selection. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

75.



76.



$y=mx+b$

$\frac{dy}{dx}=m\cdot 1x^0+0$

$\frac{dx}{dt}=m$

$\frac{dy}{dx}=2$

vertex (1,1)
Through (0,0)

$y=a(x-h)^2+k$

(h,k) vertex

$y=a(x-1)^2+1$

$0=a(0-1)^2+1$

$0=a\cdot 1+1$

$a=-1$

$y=-1(x-1)^2+1$

$y=-x^2+2x\cancel{-1+1}$

$\frac{dy}{dy}=-2x+2=$ Line

77. Sketch the graphs of $y = x^2$ and $y = -x^2 + 6x - 5$, and sketch the two lines that are tangent to both graphs. Find equations of these lines.

$y=x^2$ $\qquad$ $y=-x^2+6x-5$

$\frac{dy}{dx}=2x$ $\qquad$ $\frac{dy}{dx}=-2x+6$

$y=mx+b$

$2x_1=-2x_2+6$

$x_1=-x_2+6$

$y-y_1=m(x-x_1)$

$y-x_1^2=m(x-x_1)$

In Exercises 59–64, determine the point(s) (if any) at which the graph of the function has a horizontal tangent line. = Slope = 0 = $\frac{dy}{dx}$

59. $y = x^4 - 2x^2 + 3$

60. $y = x^3 + x$ $\quad\emptyset$

61. $y = \frac{1}{x^2}$ $\quad\emptyset$

62. $y = x^2 + 9$

63. $y = x + \sin x, \quad 0 \le x < 2\pi$

64. $y = \sqrt{3}x + 2\cos x, \quad 0 \le x < 2\pi$

$\frac{dy}{dx}=\sqrt{3}-2\sin x$

$0=\sqrt{3}-2\sin x$

$\frac{\sqrt{3}}{2}=\sin x \qquad x=\frac{\pi}{3},\frac{2\pi}{3}$

60

$y=x^3+x$

$\frac{dy}{dx}=3x^2+1$

$0=3x^2+1$

$-1 \qquad -1$

$-1=3x^2$

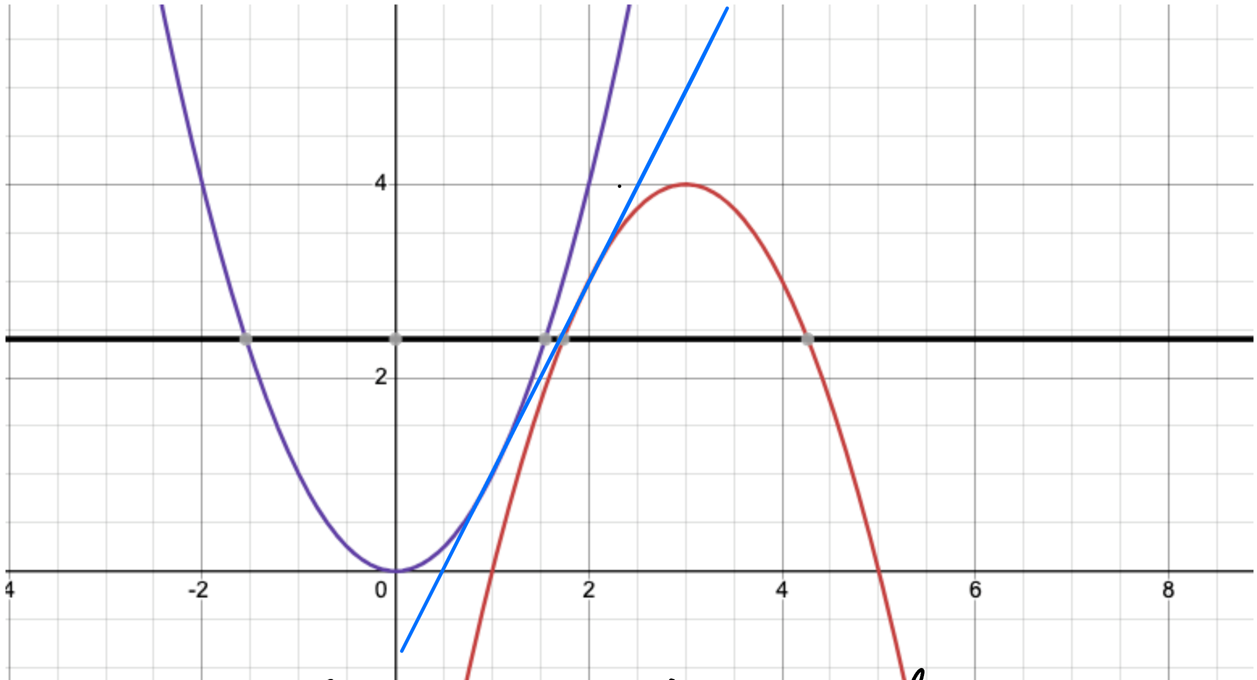
$-\frac{1}{3}=x^2$ Not gonna happen

61. $y=\frac{1}{x^2}=x^{-2}$

$\frac{dy}{dx}=-2x^{-3}=\frac{-2}{x^3}$

$0=\frac{-2}{x^3}$ Not gonna happen

$\emptyset$



completed problem in period 3 please look at period 3 notes

$$\frac{d}{dx}[e^x] = e^x$$

Product Rule

$h(x) = F(x)\cdot g(x)$

$h'(x) = F'(x)\cdot g(x) + F(x)\cdot g'(x)$

$y = \underbrace{(x^4-3x^3+6)}_{F(x)}\underbrace{(7x^4-14x^3+7x+2)}_{g(x)}$

$\frac{dy}{dx} = \underbrace{(4x^3-9x^2)}_{F'(x)}\cdot\underbrace{(7x^4-14x^3+7x+2)}_{g(x)} + \underbrace{(x^4-3x^3+6)}_{F(x)}\cdot\underbrace{(28x^3-42x^2+7)}_{g'(x)}$

$y = \underbrace{3x^2}_{F(x)} \underbrace{\sin x}_{g(x)}$

$$\frac{dy}{dx} = \overset{F'(x)\cdot g(x)}{6x \cdot \sin x} + \overset{F(x)\cdot g'(x)}{3x^2 \cdot \cos x}$$

$$h(x) = \frac{F(x)}{g(x)}$$

$$h'(x) = \frac{F'(x)\cdot g(x) - F(x)\cdot g'(x)}{(g(x))^2}$$

$$y = \frac{5x-2}{x^2+1} \Rightarrow \frac{dy}{dx} = \frac{5(x^2+1)-(5x-2)(2x)}{(x^2+1)^2}$$

Find $\frac{d^4x}{dy^4}$ for $y = 2x^3$

$y = 2x^3$

$$\frac{dy}{dx} = 6x^2 \Rightarrow \frac{d^2y}{dx^2} = 12x \Rightarrow \frac{d^3y}{dx^3} = 12 \Rightarrow \frac{d^4y}{dx^4} = 0$$

Derivatives of the Six Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$